

Introduction to the Gouy Balance

The **Gouy balance** is a classic experimental instrument used to measure the **magnetic susceptibility** of a substance. Magnetic susceptibility (denoted by the Greek letter chi, χ) quantifies how much a material will be magnetized when placed in an applied magnetic field. In simple terms, it tells us whether a substance is **diamagnetic**, **paramagnetic**, or **ferromagnetic**.

- **Paramagnetic** materials ($\chi > 0$) are weakly attracted to a magnetic field. They have unpaired electrons (e.g., O_2 , $CuSO_4$, $FeCl_3$).
- **Diamagnetic** materials ($\chi < 0$) are weakly repelled by a magnetic field. All electrons are paired (e.g., H_2O , $NaCl$, benzene).
- **Ferromagnetic** materials ($\chi \gg 0$) are strongly attracted and can retain magnetization (e.g., Fe, Co, Ni).

The Gouy balance provides a simple and effective way to distinguish between these behaviors and to quantify the effect.

Principle of Operation

The core principle is the **force experienced by a sample when placed in an inhomogeneous (non-uniform) magnetic field**.

1. **The Setup:** A long, cylindrical sample is suspended from a sensitive analytical balance. One end of the sample is positioned between the poles of a strong magnet, where the magnetic field is strongest and its gradient (the change in field strength over distance) is large. The other end is in a region of negligible magnetic field (far away from the poles).
2. **The Force:**
 - A **paramagnetic** sample is pulled *into* the strongest part of the field (it appears heavier when the magnet is turned on).
 - A **diamagnetic** sample is pushed *out of* the strongest part of the field (it appears lighter when the magnet is turned on).
3. **The Measurement:** The Gouy balance measures this apparent change in mass (Δm) of the sample when the magnetic field is applied. This change in mass is directly related to the force (F) on the sample:
 - For a paramagnet: $F = \Delta m \cdot g$ (force downward, apparent mass increase)
 - For a diamagnet: $F = -\Delta m \cdot g$ (force upward, apparent mass decrease)
where $*g*$ is the acceleration due to gravity.

Calculation of Magnetic Susceptibility

The fundamental equation that relates the measured force to the magnetic susceptibility is derived from the energy of a material in a magnetic field.

1. The Basic Force Equation:

The force on the sample in the z-direction is given by:

$$F_z = \frac{1}{2} \mu_0 \cdot \chi \cdot V \cdot \frac{d(H^2)}{dz}$$

Where:

- F_z is the measured force (in Newtons, N).
- μ_0 is the permeability of free space ($4\pi \times 10^{-7} \text{ N A}^{-2}$).
- χ is the **volume magnetic susceptibility** (dimensionless).
- V is the volume of the sample (in m^3).
- $\frac{d(H^2)}{dz}$ is the field gradient along the sample length.

Since the force is measured as an apparent change in mass, $F_z = \Delta m \cdot g$, we can write:

$$\Delta m \cdot g = \frac{1}{2} \mu_0 \cdot \chi \cdot V \cdot \frac{d(H^2)}{dz}$$

2. The Simplified Gouy Equation (for a uniform sample):

In a standard Gouy setup, the sample tube has a constant cross-sectional area (A). The volume is $V = A \cdot l$, where l is the length of the sample exposed to the field gradient. If the field at the bottom (inside the magnet) is H and the field at the top (outside) is effectively 0, the integral of the field gradient simplifies considerably.

The equation then becomes:

$$\Delta m \cdot g = \frac{1}{2} \mu_0 \cdot \chi \cdot A \cdot (H^2 - 0^2) = \frac{1}{2} \mu_0 \cdot \chi \cdot A \cdot H^2$$

Rearranging for the volume susceptibility (χ):

$$\chi = \frac{2 \cdot \Delta m \cdot g}{\mu_0 \cdot A \cdot H^2}$$

3. Converting to Molar Susceptibility (χ_m):

Chemists are more interested in the **molar magnetic susceptibility (χ_m)**, which allows for comparison between different compounds. It is related to the volume susceptibility by:

$$\chi_m = \chi \cdot \frac{M}{\rho}$$

Where:

- M is the molar mass (in kg mol⁻¹ - **note the SI units!**).
- ρ is the density (in kg m⁻³).

Since density $\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{A \cdot l}$, we can substitute and simplify the equation to a very practical form:

$$\chi_m = \frac{2 \cdot \Delta m \cdot g \cdot l \cdot M}{\mu_0 \cdot m \cdot H^2}$$

Where:

- Δm = apparent mass change (kg)
- g = acceleration due to gravity (9.81 m s⁻²)
- l = length of the sample in the field (m)
- M = molar mass (kg mol⁻¹)
- $\mu_0 = 4\pi \times 10^{-7}$ N A⁻²
- m = mass of the sample (kg)
- H = magnetic field strength at the pole faces (A m⁻¹)

Step-by-Step Calculation Example

Let's calculate the molar susceptibility for a paramagnetic salt.

Given:

- Sample: A hydrated metal salt
- Mass of sample, $m = 5.00 \times 10^{-5}$ kg (50.0 mg)
- Molar Mass, $M = 0.250$ kg mol⁻¹ (250 g mol⁻¹)
- Length of sample in tube, $l = 0.050$ m (5.0 cm)

- Magnetic field strength, $H = 1.5 \times 10^6 \text{ A m}^{-1}$ (This is a typical value for a strong electromagnet, equivalent to ~ 1.9 Tesla).
- Apparent mass change, $\Delta m = +2.00 \times 10^{-6} \text{ kg}$ (+2.00 mg) [*The + sign indicates attraction, paramagnetism*].
- **Step 1: Use the Molar Susceptibility Formula**

Step 2: Plug in all the values (in SI units)

$$\chi_m = \frac{2 \cdot (2.00 \times 10^{-6} \text{ kg}) \cdot (9.81 \text{ m/s}^2) \cdot (0.050 \text{ m}) \cdot (0.250 \text{ kg/mol})}{(4\pi \times 10^{-7} \text{ N/A}^2) \cdot (5.00 \times 10^{-5} \text{ kg}) \cdot (1.5 \times 10^6 \text{ A/m})^2}$$

Step 3: Perform the calculation

1. Calculate the numerator: $2 \times 2.00 \times 10^{-6} \times 9.81 \times 0.050 \times 0.250 = 4.9025 \times 10^{-7}$
2. Calculate the denominator:
 - $\mu_0 = 1.2566 \times 10^{-6}$
 - $m \cdot H^2 = (5.00 \times 10^{-5}) \cdot (2.25 \times 10^{12}) = 1.125 \times 10^8$
 - So, denominator = $(1.2566 \times 10^{-6}) \cdot (1.125 \times 10^8) = 141.37$
3. Final calculation: $\chi_m = \frac{4.9025 \times 10^{-7}}{141.37} \approx 3.47 \times 10^{-9}$

Result: The molar magnetic susceptibility is $\chi_m \approx 3.47 \times 10^{-9} \text{ m}^3 \text{mol}^{-1}$.

In chemistry, this value is often reported in **cgs units**. To convert from SI to cgs, multiply by $\frac{1}{4\pi} \times 10^6$.

$$\chi_m^{cgs} = (3.47 \times 10^{-9}) \times \frac{1}{4\pi} \times 10^6 \approx 2.76 \times 10^{-4} \text{ cm}^3 \text{mol}^{-1}$$

A positive value in the range of $10^{-4} \text{ cm}^3/\text{mol}$ confirms the sample is paramagnetic.

The Gouy balance is a powerful yet conceptually straightforward tool. By measuring the force on a sample in an inhomogeneous magnetic field, it provides direct access to magnetic susceptibility, a key property for understanding the electronic structure of materials. The calculation, while unit-intensive, follows directly from the physical principles of magnetism.